CLEP College Mathematics – Core Concept Cheat Sheet

04: Logic

Key Terms

- **Boolean algebra:** the algebra of logic; a symbolic representation of statements that are either true or false.
- **Conclusion:** the part of a conditional statement that comes after "then".
- **Conditional statement:** a statement that can be written in the form "if p, then q."
- **Conjunction:** two statements combined using the word "and".
- **Contrapositive:** the conditional "if not q, then not p", given the conditional statement "if p, then q."
- **Converse:** the conditional "if q, then p" given the conditional statement "if p, then q."
- **Counterexample:** used to prove a conditional statement is false; will show the hypothesis is true and the conclusion is false.
- **Disjunction:** two statements combined using the word "or".
- **Hypothesis:** the part of a conditional statement that comes after "if".
- **Inverse:** the conditional "if not p, then not q", given the conditional statement "if p, then q."
- **Logic:** the science that investigates the rules controlling reliable inference.
- **Necessary condition:** if we do not have p, then we won't have q.
- **Negation:** the opposite meaning of the original conditional statement.
- **Sufficient condition:** if we have p, we know that q must follow.
- **Truth value:** whether a conditional statement is true or false.
- **Truth table:** a table showing the truth values of the hypothesis, conclusion, and some of the related statements.
- **Universal set:** the set that contains all of the elements relevant to a given discussion.
- **Venn diagrams:** a visual organizer consisting of two or more overlapping circles.

Laws of Boolean Algebra

Some laws of Boolean algebra are:

Associative Law: $a \vee (b \vee c) = (a \vee b) \vee c$

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

Commutative Law: $a \vee b = b \vee a$

$a \wedge b = b \wedge a$

Distributive Law: $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Identify Law: $a \vee a = a$

$a \wedge a = a$

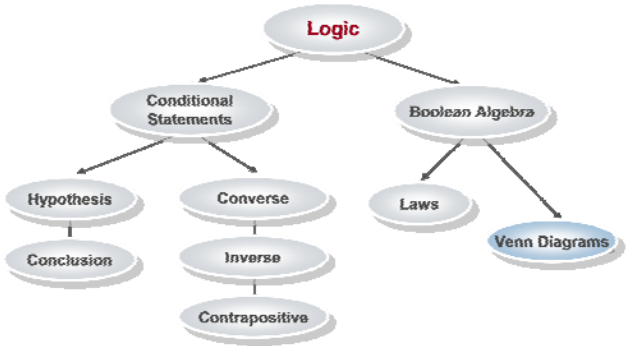
Complements Law: $a \vee \sim a = 1$

$a \wedge \sim a = 0$

Distributive Law: $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Concept Map



Constructing a Venn Diagram

To construct a Venn diagram:

1. Identify the concepts being compared.
2. List the similarities and differences of the concepts.
3. Draw and label a circle for each concept with appropriate overlapping for similarities.
4. List the common characteristics of concepts in the overlapping areas; list unique characteristics in the non-overlapping areas.

Example: Hypothesis and Conclusion

Identify the hypothesis and conclusion: If it is a weekday, then the office is open.

Solution: The hypothesis is what follows "if"; the conclusion is what follows "then".

Hypothesis → It is a weekday.

Conclusion → The office is open.

Example: Converse

Write the converse of this statement: If you are in the city of Los Angeles, then you are in the state of California.

Solution: Switch the roles of the hypothesis and conclusion to create the converse of a conditional statement.

Converse → If you are in the state of California, then you are in the city of Los Angeles.

The truth value of the converse is false.

Example: Inverse

Write the inverse of this statement: If the sum of the measures of two angles is 90°, then the two angles are complements.

Solution: Find the inverse of a conditional statement by replacing the hypothesis and conclusion with their negations.

Inverse → If the sum of the measures of two angles is not 90°, then the two angles are not complements.

The truth value of the inverse is true.

Example: Contrapositive

Write the contrapositive of this statement: If the sum of the measures of two angles is 90°, then the two angles are complements.

Solution: The contrapositive of the conditional statement "if p, then q", is the conditional "if not q, then not p."

Contrapositive → If two angles are not complements, then the sum of the measures of the two angles is not 90°.

The truth value of the contrapositive is true.

How to Use This Cheat Sheet: These are the keys related this topic. Try to read through it carefully twice then rewrite it out on a blank sheet of paper. Review it again before the exams.