

10: Graphs of Trigonometric Functions

Key Terms

- **Amplitude:** one-half of the vertical distance between the maxima and minima of a sine or cosine graph.
- **Asymptote:** any line that a function approaches closely without ever intersecting.
- **Fundamental period:** the smallest interval over which a periodic function repeats itself.
- **Horizontal translation:** a shift to the left or right of the graph of a function.
- **Key points:** points that can be used to plot the graph of a function.
- **Phase shift:** the magnitude and direction of a horizontal shift; determined by the ratio $\frac{c}{b}$.
- **Vertical asymptote:** a vertical line that a function approaches closely without ever intersecting.
- **Vertical translation:** an upward or downward shift in the graph of a function.

Key Formulas

The general forms of the equations for sine, cosine, cosecant, and secant:

- **Sine:** $y = d + a \sin (bx - c)$
- **Cosine:** $y = d + a \cos (bx - c)$
- **Tangent:** $y = d + a \tan (bx - c)$
- **Cosecant:** $y = d + a \csc (bx - c)$
- **Secant:** $y = d + a \sec (bx - c)$
- **Cotangent:** $y = d + a \cot (bx - c)$

The constants a , b , c , and d , in each equation are used to determine:

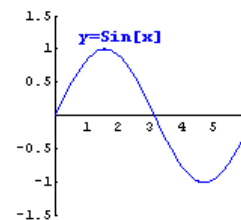
- Amplitude = $|a|$
- Two equations for period:
 - Period = $2\pi/b$ (for sin, cos, csc, sec)
 - Period = π/b (for tan and cot)
- Vertical Shift = d
- Phase Shift (Horizontal Shift) = c/b

Helpful Reminders

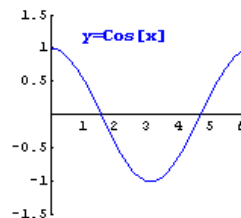
- Using key points is a valuable strategy when drawing a graph.
- List a few key points near the appropriate side of the asymptote to determine how the graph behaves as it approaches the asymptote from the given side.
- The graphs of the trigonometric functions cosine and sine do not have vertical asymptotes.
- The graphs of the trigonometric functions tangent, cotangent, secant, and cosecant have vertical asymptotes.
- The formula to find the period of cosine, sine, secant, and cosecant functions differs from the formula used to find the period of tangent and cotangent functions.
- To graph transformations of a function, first graph the basic/parent function. Then graph each transformation in a series of graphs until you reach the final graph.

Basic Graphs

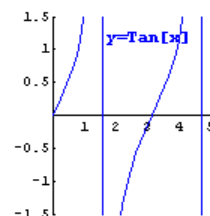
- **Sine ($y = \sin x$):** amplitude = 1, period = 2π



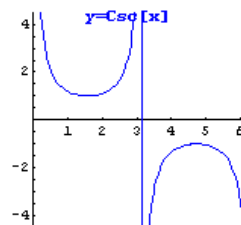
- **Cosine ($y = \cos x$):** amplitude = 1, period = 2π



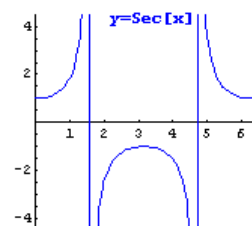
- **Tangent ($y = \tan x$):** period = π , asymptotes: $x = \{ \dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots \}$



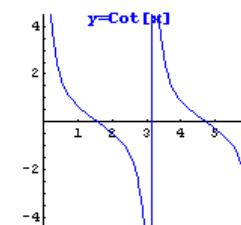
- **Cosecant ($y = \csc x$):** period = 2π , asymptotes: $x = \{ \dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots \}$



- **Secant ($y = \sec x$):** period = 2π , asymptotes: $x = \{ \dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots \}$



- **Cotangent ($y = \cot x$):** period = π , asymptotes: $x = \{ \dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots \}$



How to Use This Cheat Sheet: These are the key concepts related this topic. Try to read through it carefully twice then rewrite it out on a blank sheet of paper. Review it again before the exam.